Interval Estimation for Indonesia Democracy Index (IDI) Model Using Multivariables Spline Truncated

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Abstract

A Good democratic performance is a fundamental target of national development in the field of politics that can be measured through IDI. If IDI is modeled by using multiple linear regression yielding model with determination coefficient of 42,30% with none of significant regression parameter if tested individually by using t-test statistic, but the best IDI modeling using multivariables spline truncated regression analysis on factors influencing it having three knots (3,3,3,3,3,3) with determination coefficient (R^2) is 97,04%, with only one variable having no effect on the IDI of the six variables in the model. Interval estimation of IDI score is formed in pessimistic estimation point of view indicates Yogyakarta, Kalimantan Timur, and Kalimantan Utara experiencing degradation of democracy performance from high to moderate category, Maluku Utara is degraded from moderate to bad, while Papua Barat and Papua still in the bad category.

Keywords : Indonesia Democracy Index (IDI); Interval Estimation; Spline Truncated

1. Introduction

The wave of global democratization is like a flood that has shifted the nondemocratic regimes and replaced them with the democratic regime. From this great wave of democratization, it is needed to know the extent to which democratization has taken place including in the state of Indonesia. Indonesia needs to know the level of democratic development at the local level because its success as a democratic country will depend largely on the extent to which democracy develops and is applied throughout the province in Indonesia. Referring to the third National Development Plan of the Medium Term (RPJMN) 2015-2019, and referring to the National Long Term Development Plan (RPJPN) 2005-2025, the Government has set IDI as one of the main target of national development. Based on IDI score 2014, which is 73.04, expected in 2019, IDI score reached 75.00 (BAPPENAS 2014). There were several studies on IDI that review the factors that influence IDI, namely with time series analysis (Burkhart and Lewis-Beck 1994). The other methods which are limited to parameter estimation in parametric regression (Norris and Inglehart 2002; Drazanova 2010; Adams 2013; Högström 2013; Doko 2014; Dlamini 2015) and using the literature study (Purwanto and Syawie 2012). The problems of democracy in Indonesia can be influenced by several social, economic, and information and communication technologies. Factors affecting Indonesia's democratic level are not single. To see the magnitude of the influence of each of these factors can be done by regression analysis. Regression analysis can be done using parametric regression, nonparametric regression (Budiantara 2000; Budiantara, Suryadi, Otok, and Guritno 2006; Lestari, Budiantara, Sunaryo, and Mashuri 2012; Fernandes, Budiantara, Otok, and Suhartono 2015) or semiparametrik regression (Budiantara 2007; Budiantara, Ratnasari, Ratna, and Zain 2015; Ratnasari , Budiantara, Ratna, and Zain 2016).

In this study, to determine the relationship of IDI with the factors that influence it used using multivariables spline truncated regression. The spline truncated regression approach is capable of handling smooth data and functional characteristics of data with variable behavior on sub-intervals (Budiantara 2009). Focus of the discussion in this study is analysis of interval estimation for IDI model using multivariables spline truncated regression. Several studies to obtain interval estimation in a nonparametric regression have been done by several researchers. Among these studies are still limited to univariable nonparametric regression (Wahba 1990; Hardle 1994; Mao and Zhao 2003; Syaranamual 2011; Intansari 2016). Research on interval estimation of multivariables spline truncated regression has been done but it was limited to interval estimation for parameters of multivariables spline truncated regression (Setiawan 2017).

Multivariables spline truncated model on nonparametric regression analysis was a nonparametric regression analysis consisting of one response variable and more than one predictor variable (Budiantara 2009). If given paired data $\left(x_{1i}, x_{2i}, \ldots, x_{pi}, y_i\right)$ $i = 1, 2, \ldots, n$ and the relationship between $\left(x_{1i}, x_{2i}, \ldots, x_{pi}\right)$ and y_i is nonparametric regression multivariables model that can be written as follows :

$$y_i = f\left(x_{1i}, x_{2i}, \ldots, x_{pi}\right) + \varepsilon_i,\ i = 1, 2, \ldots, n \tag{1.1}$$

where $f\left(x_{1i}, x_{2i}, \ldots, x_{pi}\right)$ is an unknown form regression model. If the regression model $f\left(x_{1i}, x_{2i}, \ldots, x_{pi}\right)$ is assumed to be additive and approached with linear truncated spline function then we get the multivariables spline truncated model on nonparametric regression as follows.

$$y_i = \sum_{j=1}^{p} f\left(x_{ji}\right) + \varepsilon_i,\ i = 1, 2, \ldots, n \tag{1.2}$$

where

$$f\left(x_{ji}\right) = \delta_0 + \delta_{j1} x_{ji} + \sum_{l=1}^{r} \delta_{j(1+l)} \left(x_{ji} - K_{jl}\right)_+ ,$$

and

$$\left(x_{ji} - K_{jl}\right)_+ = \begin{cases} \left(x_{ji} - K_{jl}\right)_+ & , \quad x_{ji} \geq K_{jl} \\ 0 & , \quad x_{ji} < K_{jl} \end{cases} \tag{1.3}$$

with $K_{j1}, K_{j2}, \ldots, K_{jr}$ is the knot points showing the pattern of behavioral changes of the functions at different sub-intervals.

$$y_i = \delta_0 + \sum_{j=1}^{p} \left(\delta_{j1} x_{ji} + \sum_{l=1}^{r} \delta_{j(1+l)} \left(x_{ji} - K_{jl}\right)_+\right) + \varepsilon_i, \tag{1.4}$$

Equation (1.4) can be written in mathematical notation as multivariables spline truncated regression model as follows :

$$\underset{\sim}{y} = \mathbf{X(K)} \underset{\sim}{\delta} + \underset{\sim}{\varepsilon}, \quad \underset{\sim}{\varepsilon} \sim \text{IIDN}(0, \sigma^2 I) \tag{1.5}$$

The parameter estimation of multivariables spline truncated regression can be obtained from the assumption of random error and it is identic, independent and normal distributed with mean zero and variance σ^2. The probability density function of random error ε_i can be written as follows :

$$f\left(\varepsilon_i\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2} \varepsilon_i^2\right), \quad i = 1, 2, \ldots, n. \tag{1.6}$$

The likelihood function is a common probability function of the random variable $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ with $\underset{\sim}{\delta}$ as a parameter can be written as follows :

$$\begin{aligned} L\left(\underset{\sim}{\delta}\right) &= f\left(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\right), \\ &= \left(2\pi\sigma^2\right)^{-\frac{n}{2}} \exp\left(-\frac{1}{2\sigma^2}\left(\underset{\sim}{y} - \mathbf{X(K)}\underset{\sim}{\delta}\right)' \left(\underset{\sim}{y} - \mathbf{X(K)}\underset{\sim}{\delta}\right)\right). \end{aligned} \tag{1.7}$$

The parameter estimation is done by transforming the natural logarithm of equation (1.7) and by optimizing the likelihood function the following results :

$$\hat{\underset{\sim}{\delta}} = \left(\mathbf{X}'\mathbf{(K)}\mathbf{X(K)}\right)^{-1} \mathbf{X}'\mathbf{(K)} \underset{\sim}{y} \tag{1.8}$$

The point estimation for model of multivariables spline truncated regression can be obtained by substituting equation (1.8) into equation (1.5) as follows :

$$\begin{aligned}\hat{\underset{\sim}{y}} = \hat{\underset{\sim}{f}}\left(\underset{\sim}{x}\right) &= \mathbf{X(K)}\hat{\underset{\sim}{\delta}},\\ \hat{\underset{\sim}{f}}\left(\underset{\sim}{x}\right) &= \mathbf{X(K)}\left(\mathbf{X'(K)X(K)}\right)^{-1}\mathbf{X'(K)}\underset{\sim}{y},\\ &= \mathbf{A(K)}\underset{\sim}{y},\end{aligned} \tag{1.9}$$

where

$$\hat{\underset{\sim}{f}}'\left(\underset{\sim}{x}\right) = \left(\hat{f}_1\left(\underset{\sim}{x}\right)\ \hat{f}_2\left(\underset{\sim}{x}\right)\ \cdots\ \hat{f}_n\left(\underset{\sim}{x}\right)\right),$$

$$\mathbf{A(K)} = \mathbf{X(K)}\left(\mathbf{X'(K)X(K)}\right)^{-1}\mathbf{X'(K)}.$$

To find the shortest interval estimate of univariable spline truncated regression model, then assume that given univariable spline truncated regression model $y_i = f\left(x_i\right)+\varepsilon_i,\ \varepsilon_i \sim \text{IIDN}\left(0,1\right),\quad i=1,2,\ldots,n$ where $f\left(x\right)$ is a univariable spline function with as much as r knot, to obtain interval estimation of the model then can be formed a pivotal quantity (Mao and Zhao 2003) as follows :

$$T\left(x,y\right) = \frac{\hat{f}_r\left(x\right) - E\left(\hat{f}_r\left(x\right)\right)}{\sqrt{\text{var}\left(\hat{f}_r\left(x\right)\right)}} \sim \text{N}\left(0,1\right) \tag{1.10}$$

so it can be constructed the shortest estimation (1-α) × 100% for the univariable spline truncated regression model as follows:

$$\hat{f}_r\left(x\right) \pm z_{\frac{\alpha}{2}} \cdot \sqrt{\text{var}\left(\hat{f}_r\left(x\right)\right)} \tag{1.11}$$

Suppose a given matrix $\mathbf{X(K)}$ of size n x $(1+p+pr)$, column vector $\underset{\sim}{\delta}$ size $(1+p+pr)$ x n, multivariables spline truncated regression model $\underset{\sim}{y} = \mathbf{X(K)}\underset{\sim}{\delta} + \underset{\sim}{\varepsilon},\ \underset{\sim}{\varepsilon} \sim \text{IIDN}(0,\sigma^2 I)$ where σ^2 unknown, estimation of model parameters is $\hat{\underset{\sim}{\delta}} = \left(\mathbf{X'(K)X(K)}\right)^{-1}\mathbf{X'(K)}\underset{\sim}{y}$ and point estimation of multivariables spline truncated regression point is $\hat{\underset{\sim}{f}}\left(\underset{\sim}{x}\right) = \mathbf{A(K)}\underset{\sim}{y}$, where $\mathbf{A(K)}$ is a symmetric and idempotent matrix, then interval estimation for the multivariables spline truncated regression model on the *i-th* observation is :

$$P\left(\hat{f}_i\left(\underset{\sim}{x}\right) - t_{\left(\frac{\alpha}{2},\left(n-\left(1+p+pr\right)\right)\right)}\sqrt{\frac{\underset{\sim}{y}'\left(\mathbf{I}-\mathbf{A(K)}\right)\underset{\sim}{y}}{n-\left(1+p+pr\right)}\,.\mathbf{A(K)}_{ii}} \le f_i\left(\underset{\sim}{x}\right) \le \hat{f}_i\left(\underset{\sim}{x}\right) + t_{\left(\frac{\alpha}{2},\left(n-\left(1+p+pr\right)\right)\right)}\sqrt{\frac{\underset{\sim}{y}'\left(\mathbf{I}-\mathbf{A(K)}\right)\underset{\sim}{y}}{n-\left(1+p+pr\right)}\,.\mathbf{A(K)}_{ii}}\right) = 1-\alpha,\ i=1,2,\ldots,n, \tag{1.12}$$

The best model of multivariables spline truncated regression is based on the selection of the optimum knot point. One method used to have an optimal knot point is to use the Generalized Cross Validation (GCV) method (Budiantara 2000). The best spline truncated nonparametric regression model is obtained from the minimum GCV value with the following formula :

$$\text{GCV}\left(K_1,K_2,\ldots,K_r\right) = \frac{\text{MSE}\left(K_1,K_2,\ldots,K_r\right)}{\left(n^{-1}tr\left[\mathbf{I}-\mathbf{A}\left(K_1,K_2,\ldots,K_r\right)\right]\right)^2} \tag{1.13}$$

One measure of the accuracy of the model that is able to explain the relationship between predictor and response variables is to use the coefficient of determination (R^2). Generally, If the value R^2 is high, so the model is very good (Gujarati 2003).

2. Indonesia Democracy Index (IDI)

IDI is an objective and empirical measure of the condition of political democracy in Indonesia in 3 aspects, namely civil liberties, political rights, and democratic institutions. IDI aims to quantitatively measure the level of development of democracy. IDI is a tool of general check up to the democratic condition both national and provincial level. In addition, it should be emphasized that IDI is not really a tool to evaluate the performance of the government alone because the components that make up the indicators, variables and aspects of IDI not only measure the scope of government tasks alone, but at the same time also measure the growing democracy in the community (BPS 2012) .

3. Data Source, Research Variables, and Steps of Analysis

The data used in this study is secondary data that has been published by the Central Bureau of Statistics (BPS) in Statistics Indonesia 2016 and Statistics Official BPS (BRS) 2016 and publication by KEMENPPPA in Gender Based Human Development 2016. In this study, the variables used are the response variable that IDI (Y), the predictor variables, i.e Index of Human Development/IPM (X_1), Index of Gender Empowerment/IDG (X_2), Economic Growth Rate/LPE (X_3), Percentage of the Poor/PPM (X_4), Gini Ratio/GR (X_5), and Development Index of Information and Communication Technology/IPTIK (X_6). Research phase for case study of interval estimates on IDI data of Year 2015 are

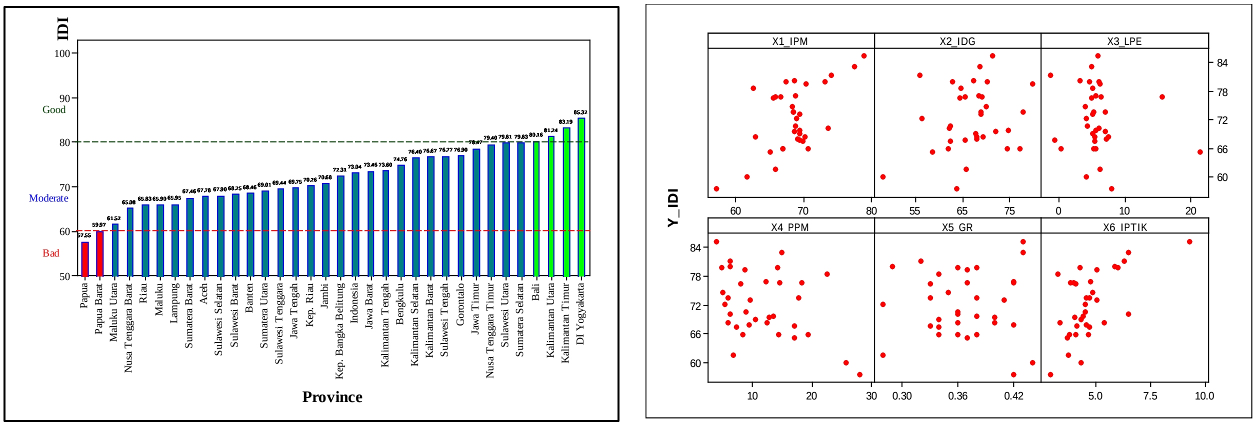
- Create descriptive analysis and scatterplot between response variables and predictor variables.
- Analyze the models using multiple linear regression
- Establishes multivariables spline truncated regression model of IDI data from the use of 1 knot point, 2 point knots, 3 knot points and a combination of knot points.
- Get the best IDI model from knot point selection with minimum GCV value and calculate the coefficient of determination.
- Obtained point estimation and interval estimation of the IDI model
- Make an interpretation from the best IDI model.

4. Result And Discussion

IDI Indonesia 2015 reached score 72.82 on a scale of 0 to 100. The achievements of Indonesia's democratic performance are still in the medium category. The classification of democracy is classified into three categories, ie good category (index> 80), moderate category (index 60 - 80), and bad category (index <60). Since formulated in 2009 until 2015, IDI performance has fluctuated. The picture of the dynamics of Indonesian democracy over the past seven years is quite unique from its fluctuations. As an embodiment of Indonesia's unique democratic development, IDI is designed to be sensitive to the ups and downs of Indonesia's democratic conditions based on fact or reality. The development of IDI Province in Indonesia is quite varied. By 2015, there are four provinces that are at the level of democratic performance which are good categorized, namely Jakarta (85,32), Yogyakarta (83,19), Kalimantan Timur (81,24), and Kalimantan Utara (80,16) presented in Fig. 1(a). In 2015 there are 28 other provinces that are in moderate performance, and there are two provinces that still categorize the poor performance of democracy, namely Papua Barat and Papua.

In addition to the deterministic factor of IDI compilers, it is necessary to consider the factors that are probabilistically affect the IDI. Factors influencing IDI (Y) in this research are IPM (X_1), IDG (X_2), LPE (X_3), PPM (X_4), GR (X_5), dan IPTIK (X_6). Descriptive analysis of each research variable can be shown as in table 1. it can be explained that IDI 2015 from as many as 34 Provinces in Indonesia has a score range from minimum 57,55 (Papua) to the highest score of 85,32 (Jakarta) and has an average IDI score of 72.12. IDI Indonesia 2015 based on Fig.1(a) has a score of 72,82. From IDI Indonesia score it can be seen that there are 18 provinces that have IDI score of Province under IDI Indonesia score, while the rest as many as 16 provinces, already have score of IDI Province above score IDI Indonesia.

ISSN 2229-5518



(a) (b)

FIGURE 1. (a) IDI Province in Indonesia Year 2015, (b) Scatter Plot Variable Response to Predictor Variables.

In the IPM factor it is seen that the achievement of human development at the provincial level varies considerably. IPM at the provincial level ranged from 57,25 (Papua) to 78,99 (Jakarta). In the IDG factor, the highest and lowest intergroup IDG spacing has a range of 29,68. IDG highest occurred in Sulawesi Utara with a value of 79,82 while the lowest is Papua Barat with a value of 48,19. In provincial LPE factor in Indonesia in 2015 there are two provinces that experienced slowdown, namely Aceh (-0,72) and Kalimantan Timur (-1,28). While the rapid LPE occurred in Nusa Tenggara Barat (21,24).

TABLE 1. Descriptive Statistics of IDI and Affecting Factors

Variable	Minimum	Maximum	Average	Standard Deviation
Y	57,55	85,32	72,12	6,75
X_1	57,25	78,99	68,58	4,17
X_2	48,19	79,82	66,88	6,49
X_3	-1,28	21,24	5,70	3,87
X_4	3,93	28,17	11,85	6,21
X_5	0,28	0,44	0,37	0,04
X_6	2,91	9,25	4,69	1,19

In the PPM factor between provinces in Indonesia, there is a high disparity. The highest PPM occurred in Papua (28,17%) and the smallest in Jakarta (3,39%). There are three provinces with PPM above 20% ie Nusa Tenggara Timur (22,61%), Papua Barat (25,82%), and Papua (28,17). In the GR factor, in 2015 the provinces are able to suppress the lowest income inequality namely Kep. Bangka Belitung and Maluku Utara (0,28) and the largest inequality occurred in Papua Barat (0,44). In the IPTIK factor, Jakarta has an almost perfect level of information and communication technology development that is 9,25 from IPTIK scale from 0 to 10. This is very different from the level of information and communication technology development that occurred in Papua of 2.91, on average only reached 4,69. Distribution of information and communication technology development in this case happened inequality.

If analyzed by multiple linear regression then obtained result that although variance error model fulfill independent characteristic, identical, and normal distribution but model have coefficient of determination only equal to 42,30%. This means that the model for IDI data containing 6 predictor variables, i.e. human development index, gender empowerment index, economic growth rate, percentage of poor population, gini coefficient, and index of information technology development and communication can explain response variable, i.e. IDI 2015 only amounted to 42.30%. Further analysis showed that the model produced by multiple linear regression although simultaneously tested obtained significant results using F-test statistic, but there is no significant regression parameter if tested individually by using t-test statistic. This indicates the presence of multicollinearity among the predictor variables.

In the scatter plot of Fig. 1(b), the initial predictive pattern of IDI relationship with IPM has different behavior at three intervals, ie interval below 67,05, interval 67,05-73,27 and interval above 73,27. At intervals below 67,05, an increase in IPM has an effect on IDI increase but slowly, at intervals between 67,05 and 73,27 increment of IPM has an effect on rapid IDI increase, and at intervals above 73,27 increases in IPM have an effect on IDI increase but with slow movement. For scatter plot results between IDI and IDG, LPE, PPM, GR, and IPTIK look less patterned. Based on this initial assumption, this research used multivariables spline truncated regression model with linear spline approach and used knot selection test consisting of one, two, three, and combination of knot point. In the nonparametric multivariable spline truncated regression was not done multicollinearity examination, because of the nonparametric multivariables spline truncated regression was part of ridge regression, that was a regression analysis used to cope with high multicollinearity. Form of accommodation to address high multicollinearity in the nonparametric multivariables spline truncated, the model $\underset{\sim}{y} = \mathbf{X(K)}\underset{\sim}{\delta} + \underset{\sim}{\varepsilon}$, with estimator $\underset{\sim}{\delta}$, i.e. $\hat{\underset{\sim}{\delta}} = \left(\mathbf{X'(K)X(K)}\right)^{-1}\mathbf{X'(K)}\underset{\sim}{y}$, always can be determined even though the matrix $\mathbf{X'(K)X(K)}$ was a nearly singular matrix. In this case, the columns of the matrix $\mathbf{X'(K)X(K)}$ were a linear combination of other columns, so the elements of $\left(\mathbf{X'(K)X(K)}\right)^{-1}$ and variance $\underset{\sim}{\delta}$, become big. This problem was solved in the nonparametric multivariables spline truncated regression by adding parameters attached to the truncated function so that matrices are always obtained $\mathbf{X'(K)X(K)}$ which was full rank and a non singular matrix, and always get estimator $\hat{\underset{\sim}{\delta}} = \left(\mathbf{X'(K)X(K)}\right)^{-1}\mathbf{X'(K)}\underset{\sim}{y}$ which has a smaller variance.

5. Interval Estimation for IDI Model Using Multivariables Spline Truncated

From the calculation of the optimum knot point, ie based on the minimum GCV value by using one, two, three, and the combination of knot points, the best model is chosen by comparing the minimum GCV value of each knot shown in Table 2. From the comparison of GCV values in Table 2, the IDI model with three knots point has the smallest GCV value to be the best model. The location of the knot point on the variable IPM (X_1) that is 60,80 (K_{11}), 69,23 (K_{12}) dan 71,89 (K_{13}), variabel IDG (X_2) yaitu 53,35 (K_{21}), 65,62 (K_{22}) dan 69,49 (K_{23}), variabel LPE (X_3) yaitu 2,40 (K_{31}), 11,13 (K_{32}) dan 13,89 (K_{33}), variabel PPM (X_4) yaitu 7,89 (K_{41}), 17,29 (K_{42}) dan 20,25 (K_{43}), variabel GR (X_5) yaitu 0,31 (K_{51}), 0,37(K_{52})dan 0,39 (K_{53}), dan variabel IPTIK (X_6) yaitu 3,95 (K_{61}), 6,40 (K_{62})dan 7,18 (K_{63}).

TABLE 2. Minimum GCV Value Comparison

Many Knot Points	Minimum GCV Value
1	32,52
2	33,32
3	**18,71**
Knot Point Combination (3,3,2,3,3,3)	23,45

The best result of IDI model with three points of knots was obtained from the parameters estimation as Table 3. or the multivariables spline truncated multivariable regression model for IDI 2015 data with estimation of model parameters in Table 3. can be written as follows :

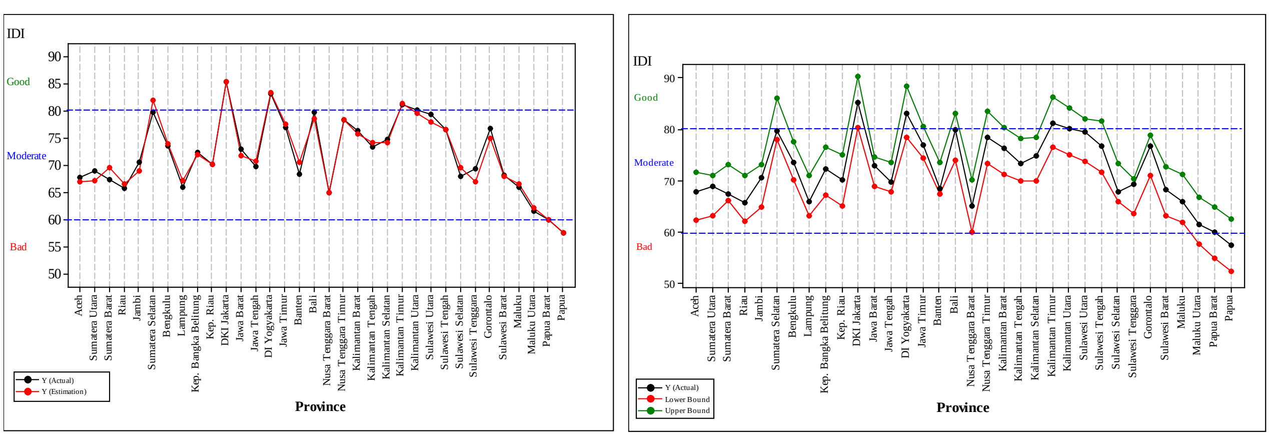
$$\begin{aligned}\hat{f}(x_{1i},...,x_{6i}) = & -3.210,24 + 29,62x_{1i} - 34,05(x_{1i} - 60,80)_+ + 6,02(x_{1i} - 69,23)_+ - 3,94(x_{1i} - 71,89)_+ + 24,67x_{2i} - 25,61(x_{2i} - 53,35)_+ + 1,57(x_{2i} - 65,62)_+ + \\ & - 0,26(x_{2i} - 69,49)_+ + 2,37x_{3i} - 2,03(x_{3i} - 2,40)_+ - 0,27(x_{3i} - 11,13)_+ - 2,94(x_{3i} - 13,89)_+ - 0,49x_{4i} + 2,21(x_{4i} - 7,89)_+ - 12,21(x_{4i} - 17,29)_+ + \\ & + 26,44(x_{4i} - 20,25)_+ + 56,10x_{5i} - 51,19(x_{5i} - 0,31)_+ - 531,03(x_{5i} - 0,37)_+ + 876,17(x_{5i} - 0,39)_+ + 45,32x_{6i} - 34,54(x_{6i} - 3,95)_+ - 216,21(x_{6i} - 6,40)_+ + \\ & + 284,49(x_{6i} - 7,18)_+ \end{aligned} \quad (5.1)$$

TABLE 3. IDI 2015 Model Parameter Estimation

Variable	Para meter	Parameter Estimation		Variabel	Para meter	Parameter Estimation	
-	δ_0	$\hat{\delta}_0$	-3.210,24				
X_1	δ_{11}	$\hat{\delta}_{11}$	29,62	X_4	δ_{41}	$\hat{\delta}_{41}$	-0,49
	δ_{12}	$\hat{\delta}_{12}$	-34,05		δ_{42}	$\hat{\delta}_{42}$	2,21
	δ_{13}	$\hat{\delta}_{13}$	6,02		δ_{43}	$\hat{\delta}_{43}$	-12,21
	δ_{14}	$\hat{\delta}_{14}$	-3,94		δ_{44}	$\hat{\delta}_{44}$	26,44
X_2	δ_{21}	$\hat{\delta}_{21}$	24,67	X_5	δ_{51}	$\hat{\delta}_{51}$	56,10
	δ_{22}	$\hat{\delta}_{22}$	-25,61		δ_{52}	$\hat{\delta}_{52}$	-51,19
	δ_{23}	$\hat{\delta}_{23}$	1,57		δ_{53}	$\hat{\delta}_{53}$	-531,03
	δ_{24}	$\hat{\delta}_{24}$	-0,26		δ_{54}	$\hat{\delta}_{54}$	876,17
X_3	δ_{31}	$\hat{\delta}_{31}$	2,37	X_6	δ_{61}	$\hat{\delta}_{61}$	45,32
	δ_{32}	$\hat{\delta}_{32}$	-2,03		δ_{62}	$\hat{\delta}_{62}$	-34,54
	δ_{33}	$\hat{\delta}_{33}$	-0,27		δ_{63}	$\hat{\delta}_{63}$	-216,21
	δ_{34}	$\hat{\delta}_{34}$	-2,94		δ_{64}	$\hat{\delta}_{64}$	284,49

Regression model of multivariables spline truncated data IDI 2015 with three point knots (3, 3, 3, 3, 3, 3) has a coefficient of determination (R^2) is 97,04%. The value of (R^2) is 97.04% can be interpreted that the model for IDI data containing 6 predictor variables, ie IPM, IDG, LPE, PPM, GR, and IPTIK can explain response variable, ie IDI 2015 is 97,04%. analysis of the interval estimation of multivariables spline truncated regression model for IDI 2015 data showing IDI score will be between the lower bound and upper bound of IDI model with 95% confidence level presented in Table 4.

The results of point estimation of IDI model show that from 34 IDI actual data, the IDI score can be accurately estimated at 6 (six) IDI Provinces (17,65%), ie IDI of Jakarta Province, Nusa Tenggara Barat, Nusa Tenggara Timur, Sulawesi Tengah, Papua Barat and Papua. While as many as 28 actual data of other provincial IDI, estimation of IDI score of Province produced is not correct. Nevertheless, with an interval estimate of 95% confidence level, it can be concluded that the IDI score of 2015 will be or contained in the interval between the lower bound and the upper bound of the model as shown in Table 4. and Fig. 2.



(a) (b)

FIGURE 2. (a) Point Estimation of IDI Model, (b) Interval Estimation of IDI Model.

Multivariables spline truncated regression model, in addition can be enabled for prediction can also be enabled for interpretation of the influence of predictor variables to response variables. In predicting IDI scores, the multivariables spline truncated regression model would be good in the function to predict if the data is within the range from the minimum sample data up to the maximum sample data. If the prediction is applied to data outside the range, the accuracy tends not to be good because of the enlarged Mean Squared Error (MSE) value. Likewise, in terms of model interval estimates. If data is taken in the minimum range of sample data and maximum sample data can be obtained the shortest interval estimation, but if the data is taken outside the sample data range the MSE value increases and the model interval estimation value becomes wider. To make predictions, suppose the highest Provincial IDI score in Indonesia, i.e. DKI Jakarta Province from the sample data IPM (X_1) equal to 78,99, IDG (X_2) equal to 71,41 , LPE (X_3) equal to 5,88 , PPM (X_4) equal to 3,93, GR (X_5) 0,43 equal to , and IPTIK (X_6) equal to 9,25, if put into the model of equation (14) taking into account the definition of the truncated function in equation (3), we get the point estimate for the IDI score of 85.32 whose value is equal to the actual data for IDI score 2015 and obtained the estimation of the interval with the lower bound of 80.29 and the upper bound of 90.35. In addition to predictive use as discussed previously, the multivariables spline truncated model can also be used as an interpretation of the effect of predictor variables on response variables. An example in the interpretation of the model can be given as follows. If it is assumed that IPM (X_1), IDG (X_2), LPE (X_3), PPM (X_4), dan GR (X_5) were constants, then the influence of IPTIK (X_6) on IDI (Y) can be indicated by the following function :

$$\hat{f}\left(x_{1i},...,x_{6i}\right)=45,32x_{6i}-34,54\left(x_{6i}-3,95\right)_{+}-216,21\left(x_{6i}-6,40\right)_{+}+284,49\left(x_{6i}-7,18\right)_{+}. \quad (5.2)$$

The truncated function in equation (5.2) contains three knot points, namely 3,95; 6,40 and 7,18 which means the influence of IPTIK on IDI has different at four different intervals. The interval formed can be written in the following function :

$$\hat{f}\left(x_{6i}\right)=\begin{cases}45,32x_{6i} & ,0\le x_{6i}<3,95\\ 10,78x_{6i}+136,43 & ,3,95\le x_{6i}<6,40\\ -205,43x_{6i}+1.520,17 & ,6,40\le x_{6i}<7,18\\ 79,06x_{6i}-522,47 & ,7,18\le x_{6i}\le 10\end{cases} \quad (5.3)$$

From the model equation (5.3) can be interpreted that if the IPTIK has a score of less than 3,95, then the IPTIK score increases greatly influence the increase in IDI score. This was indicated by the gradient of x_{6i} was positive and large values (45,32) at the IPTIK score interval of less than 3,95. The provinces that have these behaviors were Lampung, Nusa Tenggara Barat, Nusa Tenggara Timur, Gorontalo, Sulawesi Barat, Maluku Utara and Papua. This behavior was possible to catch up with the IPTIK scores of these areas, which were also known to result in an increase in democratic performance. For IPTIK scores between 3.95 and 6.40, IPTIK score increases still have a considerable effect on IDI score increase but IDI score increase at this interval is not as high as compared to IPTIK interval less than 3,95. This was indicated by the gradient of x_{6i} was positive and considerable value (10.78) at IPTIK score interval between 3,95 and 6,40. The provinces that have such behavior Aceh, Sumatera Utara, Sumatera Barat, Riau, Jambi, Sumatera Selatan, Bengkulu, Kepulauan Bangka Belitung, Jawa Barat, Jawa Tengah, Jawa Timur, Banten, Bali, Kalimantan Barat, Kalimantan Tengah, Kalimantan Selatan, Kalimantan Timur, Kalimantan Utara, Sulawesi Utara, Sulawesi Tengah, Sulawesi Selatan, Sulawesi Tenggara, Maluku dan Papua Barat. This behavior occurs in the majority of provinces in Indonesia, namely the gradual increase of IPTIK scores at intervals of 3,95 and 6,40 resulting in a gradual improvement in the performance of democracy as well. The increase in IDI score was anticlimactic when the IPTIK score was at 6,40 and 7,18 intervals. This was indicated by the gradient of x_{6i} was negative value. At this interval, increases in IPTIK scores result in a decrease in IDI scores. In this case it is possible with a high IPTIK score but its utilization does not support the portrait of democratic performance in the region. The provinces that have these behaviors were Kepulauan Riau and DI Yogyakarta. IDI scores rose sharply as the IPTIK score increased at intervals of more than 7,18 when compared with the first two intervals. This was indicated by the gradient of x_{6i} was positive and considerable value (79,06). In this case it is possible with a high IPTIK score but its utilization is consistent with the performance of democracy in the region. There is one province that has such behavior that is DKI Jakarta. For the interpretation of the effect of the other five predictor variables : IPM (X_1), IDG (X_2), LPE (X_3), PPM (X_4), and GR (X_5) to the response variable IDI (Y) can be done analogously as in the interpretation of influence IPTIC to IDI as discussed above. However, it is possible that in interpreting the effect of certain predictor variables on response variables in certain sub-intervals there is also an odd impression that is not in harmony with logic in general. For such cases, comprehensive knowledge of each research variable and the specific characteristics that occur in certain areas are required.

TABLE 4. Interval Estimation of IDI 2015 Model with 95% Confidence Level

Obs.	Province	The Actual Model	Point Estimation of Model	Interval Estimation of Model	
				Lower Bound	Upper Bound
1	Aceh	67,78	67,03	62,32	71,74
2	Sumatera Utara	69,01	67,18	63,30	71,05
3	Sumatera Barat	67,46	69,67	66,28	73,07
4	Riau	65,83	66,57	62,14	70,99
5	Jambi	70,68	69,00	64,95	73,06
6	Sumatera Selatan	*79,81*	82,05	78,08	*86,01*
7	Bengkulu	73,60	73,97	70,27	77,67
8	Lampung	65,95	67,19	63,24	71,14
9	Kep. Bangka Belitung	72,31	71,92	67,31	76,54
10	Kep. Riau	70,26	70,11	65,09	75,13
11	DKI Jakarta	85,32	85,32	80,29	90,35
12	Jawa Barat	73,04	71,78	68,91	74,66
13	Jawa Tengah	69,75	70,79	67,90	73,67
14	DI Yogyakarta	**83,19**	83,47	**78,49**	88,45
15	Jawa Timur	*76,90*	77,54	74,50	*80,59*
16	Banten	68,46	70,53	67,41	73,65
17	Bali	*79,83*	78,55	74,03	*83,07*
18	Nusa Tenggara Barat	65,08	65,08	60,05	70,11
19	Nusa Tenggara Timur	*78,47*	78,47	73,44	*83,50*
20	Kalimantan Barat	*76,40*	75,86	71,26	*80,45*
21	Kalimantan Tengah	73,46	74,10	69,92	78,28
22	Kalimantan Selatan	74,76	74,21	69,90	78,52
23	Kalimantan Timur	**81,24**	81,44	**76,64**	86,25
24	Kalimantan Utara	**80,16**	79,61	**75,15**	84,08
25	Sulawesi Utara	*79,40*	77,92	73,76	*82,07*
26	Sulawesi Tengah	*76,67*	76,67	71,64	*81,70*
27	Sulawesi Selatan	67,90	69,63	65,86	73,40
28	Sulawesi Tenggara	69,44	67,03	63,60	70,46
29	Gorontalo	76,77	75,03	71,10	78,96
30	Sulawesi Barat	68,25	68,03	63,23	72,83
31	Maluku	65,90	66,61	61,90	71,32
32	Maluku Utara	**61,52**	62,24	**57,72**	66,77
33	Papua Barat	**59,97**	**59,97**	**54,94**	65,00
34	Papua	**57,55**	**57,55**	**52,52**	62,58

From the interval estimation results, assuming that the lower bound of the IDI score is a pessimistic interval estimation of IDI score and the upper bound of IDI score is an optimistic interval estimation of IDI scores then the degraded provinces of democratic performance outcomes from the interval estimation results, necessary and important to be of concern and vigilance. Degradation in this case means that the IDI score of the interval estimation results in a pessimistic estimation of IDI scores that have performance category that is worse than the actual IDI score. Provinces that should be aware of and give special attention due to the degradation of democratic performance outcomes in the perspective of pessimistic estimation of IDI scores from high category to medium category, among others Yogyakarta, Kalimantan Timur, and Kalimantan Utara. In the probability of degradation of achievements of democratic performance at lower levels, Maluku Utara should also be wary of and give special attention due to degradation of performance achievement of democracy in view of pessimistic estimation of IDI score from moderate category to bad category. As for provinces that do not experience degradation of democratic performance outcomes, the province should give more attention because it still focuses on the performance of bad democracy both from the point of view of the actual IDI score, IDI score of point estimation, and IDI score from pessimistic estimation point of view resulting from interval estimates are Papua barat and Papua. Especially for Jakarta, the result of the interval estimate does not result in a change of democratic performance category or from the result of fixed interval estimation resulted from Jakarta as a province with high democratic performance seen from upper bound and lower bound of interval has value above 80,00. Similarly, there are 20 provinces with moderate democratic performance, from the estimation results obtained in the category of moderate democratic performance. The twenty provinces are Aceh, Sumatera utara, Sumatera barat, Riau, Jambi, Bengkulu, Lampung, Bangka Belitung Kepulauan, kepulauan Riau, Jawa Barat, Jawa Tengah, Banten, Nusa Tenggara Barat, Kalimantan Tengah, Kalimantan Selatan, Sulawesi Selatan, Sulawesi Tenggara, Gorontalo, Sulawesi Barat, and Maluku.

6. Conclussions

Based on the results and the discussion which have been done then it can be concluded that the interval estimation for IDI model using multivariables spline truncated regression as follows :

1. Analysis of IDI 2015 by using multiple linear regression yielded model with coefficient of determination equal to 42,30% with no significant regression parameter if tested individually by using t-test statistic so that model not feasible to be used .
2. The best 2015 IDI model obtained is to use three knot points on each predictor variable and the resulting coefficient of determination (R^2) of the IDI 2015 model is 97,04%, with only one variable having no effect on the IDI of the six variables in the model, so the model is feasible to use.
3. Interval estimation of IDI model is obtained shows a 95% probability that the 2015 IDI model will be contained at intervals with a certain threshold of lower bound (pessimistic estimate) and upper bound (optimistic estimate).
4. Yogyakarta, Kalimantan Timur, and Kalimantan Utara indicated degradation of high-to-moderate democratic performance outcomes in the perspective of pessimistic estimation of IDI scores. Maluku Utara indicated a degradation of performance achievement of democracy from moderate to bad in view of pessimistic estimation of IDI score. Papua Barat and Papua indicated no degradation of their democratic performance performance, but still focused on poor performance of democracy performance both from the point of view of actual IDI score, IDI score of point estimation, and IDI score from pessimistic estimation point.

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